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# 2-factors and independent sets on claw-free graphs


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## Abstract

In this paper, we show that if $G$ is an $l$-connected claw-free graph with minimum degree at least three and $l \in \{2, 3\}$, then for any maximum independent set $S$, there exists a 2-factor in which each cycle contains at least $l - 1$ vertices in $S$.


## 1 Introduction

In this paper, we consider finite graphs. If no ambiguity can arise, we denote simply the order $|G|$ of $G$ by $n$, the minimum degree $\delta(G)$ by $\delta$ and the independence number $\alpha(G)$ by $\alpha$. All notation and terminology not explained in this paper is given in [4] or [1].

A *2-factor* of a graph $G$ is a spanning 2-regular subgraph of $G$. Choudum and Paulraj [3] and Egawa and Ota [5] independently showed that every claw-free graph with $\delta \geq 4$ has a 2-factor. For the upper bound of the number of cycles in 2-factors, Broersma, Paulusma and Yoshimoto [2] proved that a claw-free graph with $\delta \geq 4$


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has a 2-factor with at most $\max\left\{\frac{n-3}{\delta-1}, 1\right\}$ cycles. This upper bound is almost best possible. (See [17].) Faudree et al. [6] studied a pair of a maximum independent set and a 2-factor of a claw-free graph $G$ which together dominate $G$ and showed that if $G$ is a claw-free graph with $\delta \geq \frac{2n}{\alpha} - 2$ and $n \geq \frac{3\alpha^3}{2}$, then for any maximum independent set $S$, $G$ has a 2-factor with $\alpha$ cycles such that each cycle contains exactly one vertex in $S$. The following problems were posed in their article.

**Conjecture A** ([6]). *Let $G$ be a claw-free graph.*

1. *If $\delta \geq \frac{n}{\alpha} \geq 5$, then there exist a maximum independent set $S$ and a 2-factor with $\alpha$ cycles such that each cycle contains a vertex of $S$.*

2. *If $\delta \geq \alpha + 1$, then for any maximum independent set $S$, there exists a 2-factor with $\alpha$ cycles such that each cycle contains a vertex in $S$.*

In this paper, we study 2-factors which just separate a given maximum independent set $S$, i.e., we require that every cycle contains at least one vertex of $S$, and so the number of cycles in a 2-factor can be smaller than $\alpha$. This question was posed by Kaiser when the third author gave a lecture at University of West Bohemia. However, in general still we need the condition $\delta \geq n/\alpha$ because for any positive integer $\delta$ with $\frac{n}{\alpha} - \frac{1}{2\delta} < \delta < \frac{n}{\alpha}$, there exists an infinite family of line graphs with minimum degree $\delta$ whose every 2-factor contains more than $\alpha$ cycles (see [6], [17]). However 2-connectivity decreases the lower bound of minimum degrees. Our main result of this paper is the following.

**Theorem 1.** *If $G$ is an $l$-connected claw-free graph with $\delta \geq 3$ and $l \in \{2, 3\}$, then for any maximum independent set $S$, $G$ has a 2-factor such that each cycle contains at least $l - 1$ vertices in $S$.*

We will show this in Section 2. Since a 3-connected claw-free graph has a 2-factor in which each cycle contains at least two vertices in a given maximum independent set by Theorem 1, the number of the cycles in the 2-factor is at most $\frac{\alpha}{2}$. It is well known that the independence number of a claw-free graph is at most $\frac{2n}{\delta+2}$ (for instance, see [6]), and so we obtain the following.

**Corollary 2.** *A 3-connected claw-free graph has a 2-factor with at most* $\frac{\alpha}{2} \leq \frac{n}{\delta+2}$ *cycles.*

Finally we give some additional definitions and notation. A connected graph in which every vertex has even degree is called a *circuit*. A subgraph $D$ is *dominating* a graph $G$ if $G - V(D)$ is edgeless. The degree of a vertex $u$ in $G$ is denoted by $d_G(u)$ and we denote the set of all the vertices of degree at least $k$ in $G$ by $V_{\geq k}(G)$, and $V_k(G) = \{d_G(u) = k \mid u \in G\}$. The *edge-degree* of an edge $xy$ is defined as $d_G(x) + d_G(y) - 2$. An edge subset $E_0$ is called *independent* if no pair of edges in $E_0$ are adjacent. We denote the subgraph induced by the vertex set of a subgraph $B$ in $G$ by $G[B]$. A graph $G$ is *essentially* $k$-edge-connected if for any edge set $E_0$ of at most $k-1$ edges, $G \setminus E_0$ contains at most one component with edges.

## 2 Proof of Theorem 1

Let $G_0$ be an $l$-connected claw-free graph with $\delta \geq 3$ and $l \in \{2, 3\}$ and $S_0$ be any maximum independent set of $G_0$. We look for a 2-factor in $G_0$ in which each cycle contains at least $l-1$ vertices in $S_0$.

We use Ryjáček closure of a claw-free graph $G$ which is defined as follows: for each vertex $x$ of $G$, $N_G(x)$ induces a subgraph $G[N_G(x)]$ with at most two components; otherwise there is an induced claw. If $G[N_G(x)]$ has two components, both of them must be cliques. In the case that $G[N_G(x)]$ is connected, we add edges joining all pairs of nonadjacent vertices in $N_G(x)$. The *closure* $cl(G)$ of $G$ is a graph obtained by recursively repeating this operation, as long as this is possible. Ryjacek [15] showed that the closure $cl(G)$ is uniquely determined and $G$ is hamiltonian if and only if $cl(G)$ is hamiltonian.

Ryjáček, Saito and Schelp [16, Theorem 4] proved that for any mutually vertex-disjoint cycles $D_1, \ldots, D_p$ in $cl(G)$, a claw-free graph $G$ has mutually vertex disjoint cycles $C_1, \ldots C_q$ with $p \geq q$ such that $\bigcup_{i=1}^{p} V(D_i) \subset \bigcup_{j=1}^{q} V(C_j)$. By modifying the proof, easily we can improve this result as follows:

**Lemma 3.** *If $G$ is a claw-free graph and $D_1, \ldots, D_p$ are mutually vertex-disjoint cycles in $cl(G)$, then $G$ has mutually vertex disjoint cycles $C_1, \ldots C_q$ with $p \geq q$ such that for each $D_i$, there exists $C_j$ such that $V(D_i) \subset V(C_j)$.*

If $cl(G_0)$ has a 2-factor $\bigcup_{i=1}^{p} D_i$ in which each cycle $D_i$ contains at least $l-1$ vertices in $S_0$, then by the above lemma, $G_0$ has a vertex disjoint cycles $C_1, \ldots, C_q$ such that for each $D_i$, there exists $C_j$ such that $V(D_i) \subset V(C_j)$. Since

$$|C_j \cap S_0| \geq |D_i \cap S_0| \geq l-1 \text{ and } \bigcup_{i=1}^{p} V(D_i) = V(cl(G_0)) = V(G_0),$$

$\bigcup_{j=1}^{q} C_j$ is a required 2-factor of $G_0$. We rephrase moreover the above statement using the following result.

**Lemma B** (Ryjáček [15])**.** *For any claw-free graph $G$, there exists a triangle-free graph $H$ such that $L(H) = cl(G)$.*

Let $H_0$ be a triangle-free graph such that $L(H_0) = cl(G_0)$. By the above facts, for Theorem 1, it is sufficient to show that:

$$\begin{array}{c} L(H_0) \textit{ has a 2-factor in which each cycle contains} \\ \textit{at least } l-1 \textit{ vertices in } S_0. \end{array} \tag{1}$$

Let $H$ be a graph and $\mathcal{D}$ a set of mutually edge-disjoint circuits and stars in $H$. If every star has at least three edges and every edge in $E(G) \setminus \bigcup_{D \in \mathcal{D}} E(D)$ is incident to a circuit in $\mathcal{D}$, then $\mathcal{D}$ is called a *system that dominates* $H$. Gould and Hynds [11] showed that the line graph $L(H)$ has a 2-factor with $c$ cycles if and only if there exists a system that dominates $H$ with $c$ elements. Hence, we look for a system that dominates $H_0$ such that the corresponding 2-factor of $L(H_0)$ satisfies (1).

The set in $H_0$ corresponding to $S_0$ is an edge set. We denote the edge set also by $S_0$. Notice that $S_0$ is independent in $G_0$, but $S_0$ is not always independent in $L(H_0) = cl(G_0)$, i.e.,

$$S_0 \textit{ is possibly not independent in } H_0.$$

In either case, it is sufficient to prove the following claim for (1) because the set of hamilton cycles of $L(F_1), \ldots, L(F_p)$ constructs a desired 2-factor of $L(H_0)$.

**Claim 1.** *There exist edge-disjoint subgraphs $F_1, \ldots, F_p$ in $H_0$ such that*

1. $\bigcup_{i=1}^{p} E(F_i) = E(H_0)$,
2. $L(F_i)$ *is hamiltonian for all* $i \leq p$ *and*
3. $F_i$ *contains at least* $l - 1$ *edges in* $S_0$ *for all* $i \leq p$.

**Proof of Claim 1.**

Because $L(H_0)$ is $l$-connected and has minimum degree at least three, $H_0$ is essentially $l$-edge-connected and the minimum edge-degree of $H_0$ is at least three. Since $l \in \{2, 3\}$, we can use the following lemma.

**Lemma C** (Yoshimoto [17]). *If $H$ is an essentially 2-edge-connected graph with minimum edge-degree at least three, then there exists a system $\mathcal{D}$ that dominates $H$ such that every vertex in $V_{\geq 3}(H - V_1(H))$ is in a circuit in $\mathcal{D}$.*

Let $\mathcal{D}_0 = \{B_1, \ldots, B_p\}$ be such a system that dominates $H_0$ such that the number $p$ of the elements in the system is smallest.

Suppose there exists a star $B_i$ in $\mathcal{D}_0$. If the center of $B_i$ is in $V_{\geq 3}(H_0 - V_1(H_0))$, then there exists a circuit $B_j \in \mathcal{D}_0$ which passes through the center. As every edge in $B_i$ is dominated by $B_j$, we can remove $B_i$ from $\mathcal{D}_0$. This contradicts the assumption of $\mathcal{D}_0$, and hence the center of $B_i$ is in $V_2(H_0 - V_1(H_0))$. Since the minimum edge-degree is at least three,

$$\text{a star } B_i \in \mathcal{D}_0 \text{ contains at least two pendant edges of } H_0. \tag{2}$$

Furthermore,

$$\text{if } H_0 \text{ is essentially 3-edge-connected, then there is no star in } \mathcal{D}_0. \tag{3}$$

We construct desired subgraphs $F_1, \ldots, F_p$ from $\mathcal{D}_0$ by distributing edges which are not used in $\bigcup_{i=1}^{p} B_i$. At first we modify all stars in $\mathcal{D}_0$. Let $B'_i$ be the star which is constructed by all the pendant edges of $H_0$ in $B_i$. For a circuit $B_i \in \mathcal{D}_0$, we let $B'_i = B_i$ and $\mathcal{D}'_0 = \{B'_1, \ldots, B'_p\}$.

For each $B'_i \in \mathcal{D}'_0$, let

$$S_i = E(H_0[B'_i]) \cap S_0 \text{ and } \widetilde{S} = S_0 \setminus \bigcup_{i=1}^{p} S_i$$

and

$$T_i \textit{ be a maximum independent edge set of } H_0[B'_i] - V(S_i),$$

where $V(S_i) = \{x, y \mid xy \in S_i\}$. Notice that there is a natural bijection between $V(G_0)$ and $E(H_0)$. Since the vertex subset in $G_0$ corresponding to $\bigcup_{i=1}^{p}(S_i \cup T_i)$ is independent,

$$\textit{for any edge } f \in \bigcup_{i=1}^{p} T_i, \textit{ there exists an edge } e \in \widetilde{S} \textit{ adjacent to } f; \tag{4}$$

otherwise the vertex subset in $G_0$ corresponding to $\widetilde{S} \cup \bigcup_{i=1}^{p} S_i \cup \{f\}$ is an independent set of $G_0$ which is larger than $S_0$.

Let $R$ be the bipartite graph with partite sets $\bigcup_{i=1}^{p} T_i$ and $\widetilde{S}$ obtained by joining all $f \in \bigcup_{i=1}^{p} T_i$ and $e \in \widetilde{S}$ which are adjacent in $H_0$. By (4),

$$d_R(f) \geq 1 \text{ for all } f \in \bigcup_{i=1}^{p} T_i.$$

As $\bigcup_{i=1}^{p} T_i$ is independent in $H_0$,

$$d_R(e) \leq 2 \text{ for all } e \in \widetilde{S}.$$

We can show that $R$ has a matching covering $\bigcup_{i=1}^{p} T_i$. Let $R_j$ be a component of $R$ containing an element of $\bigcup_{i=1}^{p} T_i$ and $R'_j$ a spanning tree of $R_j$. If every $e \in \widetilde{S} \cap R_j$ has degree two in $R'_j$, then $|(\bigcup_{i=1}^{p} T_i) \cap R_j| = |\widetilde{S} \cap R_j| + 1$. Since $R_j$ is a component in $R$, there is no edge joining $(\bigcup_{i=1}^{p} T_i) \cap R_j$ and $\widetilde{S} \setminus (\widetilde{S} \cap R_j)$ in $R$. Therefore, the vertex subset in $G_0$ corresponding to $\big(S_0 - \widetilde{S} \cap R_j\big) \cup \big((\bigcup_{i=1}^{p} T_i) \cap R_j\big)$ is independent and the order is $|S_0| + 1$. This contradicts the maximality of $S_0$, and so $R'_j$ contains an end $e_j \in \widetilde{S} \cap R_j$.

We give a direction to $E(R'_j)$ from the root $e_j$ to leaves. For any $f \in (\bigcup_{i=1}^{p} T_i) \cap R_j$, there exists only one $e_f \in \widetilde{S} \cap R_j$ which is the origin of $f$ on this direction. We define a mapping $\phi : \bigcup_{i=1}^{p} T_i \to \widetilde{S}$ by $\phi(f) = e_f$. As the degree of $\phi(f) \in \widetilde{S}$ is at most two, $\phi$ is a one-to-one correspondence, and so $\{fe_f : f \in \bigcup_{i=1}^p T_i,\ e_f = \phi(f)\}$ is a matching of $R$ covering $\bigcup_{i=1}^p T_i$.

The set $S_0$ can be partitioned into the following mutually disjoint subsets:

$$\bigcup_{i=1}^{p} S_i,\ \bigcup_{i=1}^{p} \phi(T_i) \text{ and } \widetilde{S} \setminus (\bigcup_{i=1}^{p} \phi(T_i)).$$

Using this partition, we distribute edges in $S_0$ to $\mathcal{D}'_0$, i.e., we define a mapping $\varphi : S_0 \to \mathcal{D}'_0$ as follows:

1. For $e \in S_i$ $(1 \le i \le p)$, we define $\varphi(e) := B'_i$.

2. For $e \in \phi(T_i)$ $(1 \le i \le p)$, we define $\varphi(e) := B'_i$.

3. For $e \in \widetilde{S} \setminus \bigcup_{i=1}^p \phi(T_i)$, there exists $B'_i \in \mathcal{D}'_0$ such that $e$ is incident to $B'_i$ since $\mathcal{D}_0$ is a system that dominate $H_0$. We let $\varphi(e) := B'_i$ for arbitrary $B'_i$ containing an end of $e$.

Because $S_i \cup \phi(T_i) \subset \varphi^{-1}(B'_i)$ and $\phi$ is a one-to-one correspondence,

$$|\varphi^{-1}(B'_i)| \ge |S_i| + |\phi(T_i)| = |S_i| + |T_i|$$

for any $B'_i \in \mathcal{D}'_0$. Therefore $\varphi^{-1}(B'_i)$ contains at least $|S_i|+|T_i|$ edges in $S_0$. If $B'_i$ is a star, then $S_i = E(B'_i) \cap S_0$ and so $|S_i|+|T_i| = 1$. If $B'_i$ is a circuit, then $|S_i|+|T_i| \ge 2$ because $H_0$ is triangle-free. Therefore,

$$\begin{cases} \textit{if } B'_i \textit{ is a star,} & \varphi^{-1}(B'_i) \textit{ contains one edge in } S_0 \\ \textit{if } B'_i \textit{ is a circuit,} & \varphi^{-1}(B'_i) \textit{ contains at least two edges in } S_0. \end{cases} \tag{5}$$

Now we divide our argument into two cases.

**Case 1.** $l = 3$, i.e., $H_0$ is essentially 3-edge-connected.

In this case, there is no star in $\mathcal{D}_0$ by (3), and so $\mathcal{D}'_0 = \mathcal{D}_0$. Since $\mathcal{D}_0$ is a system that dominates $H_0$, every edge in $E(H_0) \setminus \bigcup_{i=1}^p E(B'_i)$ is incident to some $B'_i$ in $\mathcal{D}'_0$. We define a mapping $\psi$ from

$$\widetilde{E} = E(H_0) \setminus (\bigcup_{i=1}^{p} E(B'_i) \cup S_0)$$

to $\mathcal{D}_0'$ as follows: for all $e \in \widetilde{E}$, let $\psi(e) := B_i'$ for arbitrary $B_i'$ containing an end of $e$.

Let

$$F_i = B_i' \cup \varphi^{-1}(B_i') \cup \psi^{-1}(B_i').$$

Obviously $F_1, \dots, F_p$ are mutually edge-disjoint and $\bigcup_{i=1}^{p} E(F_i) = E(H_0)$. Since every edge in $F_i$ is on the circuit $B_i$ or incident to $B_i$, $F_i$ has a dominating circuit. Harary and Nash-Williams [13] showed that $L(H)$ is hamiltonian if and only if $H$ has a dominating circuit or $H$ is a star with at least three edges. Therefore $L(F_i)$ is hamiltonian. By (5), every $F_i$ contains at least two edges in $S_0$, and hence $F_1, \dots, F_p$ are desired subgraphs.

**Case 2.** $l = 2$, i.e., $H_0$ is essentially 2-edge-connected.

In this case, $\mathcal{D}_0'$ includes stars, and hence we have to care a star with less than three edges because such stars do not induce cycles in $L(H_0)$. So, we distribute edges in $\widetilde{E} = E(H_0) \setminus \left(\bigcup_{i=1}^{p} E(B_i') \cup S_0\right)$ to $\mathcal{D}_0'$ such that every star contains at least three edges. Notice that any star $B_i' \in \mathcal{D}_0'$ contains at least two edges by (2).

Suppose $B_i'$ is a star in $\mathcal{D}_0'$ with $|E(B_i')| = 2$. If $S_i = \emptyset$, then there is $f \in T_i$, and so $\phi(f) \in \varphi^{-1}(B_i')$. Since $\phi(f) \notin E(B_i')$,

$$B_i' \cup \varphi^{-1}(B_i') \textit{ contains at least three edges.} \tag{6}$$

Therefore it is not necessary to distribute an edge in $\widetilde{E}$ to $B_i'$ if $S_i = \emptyset$. However, in the case that $|E(B_i')| = 2$ and $S_i \neq \emptyset$, we have to distribute an edge in $\widetilde{E}$ to $B_i'$.

Let $\mathcal{T}$ be the set of all stars $B_i' \in \mathcal{D}_0'$ with $|E(B_i')| = 2$ and $S_i \neq \emptyset$. The following claim implies that there exist two edges in $\widetilde{E}$ incident to the center of $B_i' \in \mathcal{T}$.

**Claim 2.** *For all $B_i' \in \mathcal{T}$, there is no edge in $S_0 \setminus S_i$ incident to $B_i'$.*

*Proof.* Suppose that there exists an edge $e_1 \in S_0$ such that $e_1 \notin E(B_i')$ and $e_1$ is incident with the center of $B_i'$. Let $e_2 \in S_i$, let $e_3 \in E(B_i') - \{e_2\}$, and let $e_4$ be the edge incident with the center of $B_i'$ other than $e_1, e_2, e_3$. In the graph $cl(G_0)$, four vertices $e_1, e_2, e_3, e_4$ form a clique. However $e_1 e_2 \notin E(G_0)$ because $e_1, e_2 \in S_0$ and

$S_0$ is independent on $G_0$. Thus, the edge $e_1e_2$ of $cl(G_0)$ is added through the closure operation. This implies that $N_{G_0}(e_2) \subset \{e_3, e_4\}$, which contradicts that $\delta \geq 3$. Thus, Claim 2 holds. $\square$

Let $K$ be the bipartite graph with partite sets $\widetilde{E}$ and $\mathcal{T}$ obtained by joining all $f \in \widetilde{E}$ and $B'_i \in \mathcal{T}$ if $f$ is incident to the center of $B'_i$. Since for any $B'_i \in \mathcal{T}$, the center of $B'_i$ is in $V_2(H_0 - V_1(H_0))$, we have $d_K(B'_i) = 2$ by Claim 2. Since $d_K(f) \leq 2$ for all $f \in \widetilde{E}$, the bipartite graph $K$ has a matching $M$ covering $\mathcal{T}$ by Hall's Theorem. Using $M$, we define $\psi : \widetilde{E} \to \mathcal{D}'_0$ as follows: let $f \in \widetilde{E}$. If $f$ is used in $M$, we define $\psi(f) := B'_i$ where $fB'_i \in M$; otherwise let $\psi(f) := B'_i$ for arbitrary $B'_i$ containing an end of $f$. Because the matching $M$ covers $\mathcal{T}$, we have $\psi^{-1}(B'_i) \neq \emptyset$ for any $B'_i \in \mathcal{T}$. Since $\psi^{-1}(B'_i) \cap E(B'_i) = \emptyset$, $B'_i \cup \psi^{-1}(B'_i)$ contains at least three edges.

Let

$$F_i = B'_i \cup \varphi^{-1}(B'_i) \cup \psi^{-1}(B'_i)$$

for $1 \leq i \leq p$. Obviously $F_1, \ldots, F_p$ are mutually edge-disjoint and $\bigcup_{i=1}^{p} E(F_i) = E(H_0)$. Since every star $F_i$ contains at least three edges, $L(F_i)$ is hamiltonian for all $F_i$ by the theorem of Harary and Nash-Williams [13]. As $F_i$ contains an edge in $S_0$ by (5), $F_1, \ldots, F_p$ are desired subgraphs. The proof of Claim 1 is completed.

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