RIMS Workshop 2023

Analytic Number Theory and Related Topics

Organizers: Yu Yasufuku (Nihon University)
Maki Nakasuji (Sophia University / Tohoku University)

Date: Tuesday, October 10, 2023 ∼ Friday, October 13, 2023
Place: Room 420, RIMS, Kyoto University, Japan
Format: Hybrid (On-site and via Zoom Meeting)

Abstracts

Tuesday, October 10

09:50–10:50 Koichi Kawada (Iwate University)

On the Waring–Goldbach problem — techniques developed in the current century

We denote by $H(k)$ the least number s such that every sufficiently large integer satisfying a "necessary" congruence restriction can be written as the sum of s k-th powers of primes. Evaluation of $H(k)$ is a central topic in the research of the Waring-Goldbach problem, and a number of papers have been devoted to this theme after the celebrated work of Vinogradov in 1937 on the ternary Goldbach problem. I shall talk on techniques and results in this area published since 2001 mainly.

11:05–11:35 Takashi Nakamura (Tokyo University of Science)

L-functions with Riemann's functional equation and the Riemann hypothesis

Let χ_4 be the non-principal Dirichlet character mod 4 and $L(s,\chi_4)$ be the Dirichlet L-function associated with χ_4, and put $R(s) := s4^s L(s+1,\chi_4)+\pi L(s-1,\chi_4)$. In this talk, we show that the function $R(s)$ has the Riemann's functional equation and its zeros only at the non-positive even integers and complex numbers with real part $1/2$. We also give other L-functions that have the same property.

11:50–12:20 Atsushi Katsuda (Kyushu University)

Prime numbers and Prime closed geodesics:Similarity and Differences

As is well known that, since Selberg's celebrated works, the prime geodesic theorem for compact manifolds with negative curvature is a geometric analogue of the prime number theorem. This similarity between number theory and geometry also holds true for the Chebotarev-type density theorems of finite extensions. However, as for infinite extensions, the situations are quite different. In this talk, we shall discuss the latter cases around our recent works for nilpotent extensions in geometry. The more detailed abstract is available at `https://rb.gy/i3va5`

13:40–14:00 Eisuke Otsuka (Tohoku University)

On iterated integrals on some specific algebraic curves of degree 2

Each multiple zeta value can be represented as an iterated integral of $\mathbb{Q}$-rational differential forms on $\mathbb{P}^1(\mathbb{C}) \setminus \{0,1,\infty\}$. In this talk, we consider the special values given by iterated integrals on some specific algebraic curves over $\mathbb{Q}$ of degree 2 and explain their basic properties. In particular, by considering the relationship between our special values and colored multiple zeta values, we give their motivic interpretations and explain its application.

This workshop is partially supported by RIMS and JSPS KAKENHI Grant Numbers 19K03412, 22K03274, 22K03263.

14:05–14:35 Shin-ichi Yasutomi (Toho University)

On continued fractions that converge simultaneously under the topology of the real numbers and the p-adic topology

Let p be a prime number and K be a field with embeddings into $\mathbb{R}$ and $\mathbb{Q}_p$. We propose a continued fraction expansion algorithm that can be expected to converge simultaneously for both $\mathbb{R}$ and $\mathbb{Q}_p$ for numbers in K. This algorithm converges with respect to the p-adic topology, and rational numbers have a finite continued fraction expansions. In the case of $p = 2$ and if K is a quadratic field, the numbers in K converge in $\mathbb{R}$, and their continued fraction expansions are eventually periodic. For an element α in K, let p_n/q_n denote the n-th convergents. There exist constants u_1 and u_2 in $\mathbb{R}_{>0}$ with $u_1 + u_2 = 2$, and constants C_1 and C_2 in $\mathbb{R}_{>0}$, satisfying :$|\alpha - p_n/q_n| < C_1/|q_n|^{u_1}$ and $|\alpha - p_n/q_n|_2 < C_2/|q_n|^{u_2}$. Here, $|\cdot|_2$ represents the 2-adic distance. For prime numbers $p > 2$, we present numerical experiences.

14:50–15:20 Wataru Takeda (Tokyo University of Science)

Brocard–Ramanujan problem for norm forms over radical fields

The Brocard-Ramanujan problem, which is an unsolved problem in number theory, is to find integer solutions (x, n) of $x^2 - 1 = n!$. Many analogs of this problem are currently being considered. As one example, it is known that there are at most only finitely many algebraic integer solutions (x, n), up to a unit factor, to the equations $N_K(x) = n!$, where N_K are the norm of number fields K. In this talk, we construct infinitely many radical number fields K such that $N_K(x) = n!$ has at least 22 solutions for positive integers n.

15:35–16:05 Tomohiro Yamada (Kobe University)

Lehmer's totient problem

Let $\varphi(N)$ denote Euler's totient function and $\omega(N)$ denote the number of distinct prime factors of a positive integer N. We show that if $N \pm 1 = M\varphi(N)$, then $M < 15.76515 \log\log\log N$ and $M < 16.03235 \log\log\omega(N)$, together with similar results for the unitary totient function, Dedekind function, and the sum of unitary divisors.

16:20–16:50 Toshiki Matsusaka (Kyushu University)

Curious congruences for cyclotomic polynomials

We promote the recent research by Akiyama and Kaneko on the higher-order derivative values $\Phi_n^{(k)}(1)$ of the cyclotomic polynomials. This talk focuses on Lehmer's explicit formula of $\Phi_n^{(k)}(1)/\Phi_n(1)$ as a polynomial of the Euler and Jordan totient functions over $\mathbb{Q}$. Then we prove Akiyama–Kaneko's conjecture that the polynomials have a specific simple factor. This is a joint work with Genki Shibukawa.

Wednesday, October 11

09:30–10:30 Michael Coons (California State University, Chico)

Towards a characterization of regular sequences

In the mid-2000s, Adamczewski and Buguead proved the Cobham-Loxton-van der Poorten conjecture by using the subspace theorem to show that any automatic number (a number whose base expansion is given by an automatic sequence) is either rational or transcendental. About 10 years later, again using the subspace theorem, Bell, Bugeaud, and Coons (the speaker), extended this result to regular sequences—a (possibly unbounded) generalization of automatic sequences. In this talk, I will discuss a further characterization of regular sequences by defining an associated measure, the so-called ghost measure, which is governed by the underlying properties of a related finite set of matrices. Given time, relationships to symbolic dynamics and fractal geometry will also be discussed.

10:45–11:15 Alan Filipin (University of Zagreb)

On the Fibonacci and Lucas numbers as products of three repdgits

Repdigit in base g is a positive integer that has only one digit in its base g expansion, i.e. a number of the form $a(g^m - 1)/(g - 1)$, for some positive integers $m \geq 1$, $g \geq 2$ and $1 \leq a \leq g - 1$. In this talk, we investigate all Fibonacci and Lucas numbers which can be expressed as products of three repdigits in base g. As illustration, we consider the case $g = 10$ where we prove that the numbers 144 and 18 are the largest Fibonacci and Lucas numbers which are expressible as products of three repdigits respectively. The main method in our proof is Baker's theory on linear forms in logarithms of algebraic numbers. This is joint work with Kouessi Norbert Adédji and Alain Tobé.

11:30–12:00 Noriko Hirata-Kohno (Nihon University)

Effective method and applications in Diophantine problems

We talk about a collaboration with Shanta Laishram and Makoto Kawashima. Fix rational integers $a, b \geq 2$. Let $1 \leq d \in \mathbb{Z}$. Consider the d-th power a^d of a and the digit expansion of a^d in the base b. Denote the sum of the digits in the base b by $s_{a,b} := s_b(a^d) = \sum_{k=0}^{n} x_k$ where $a^d = x_0 + x_1 \cdot b + \cdots + x_n \cdot b^n$. Whenever $\frac{\log a}{\log b} \notin \mathbb{Q}$, then the exponent d takes only finitely many possible integers and can be explicitly determined depending on $s_{a,b}$, by using a work of C. Stewart. It concerns an effective version of the result originally dealt by H. G. Senge and E. G. Strauss, as well as a question posed by Y. Odaka.

Further, not only such a power, the statement can be generalized to numbers satisfying a polynomial-exponential equation, including those given by a linear recurrence sequence whose characteristic polynomial has a dominant root.

We also discuss other related joint works relying on effective method of the same nature.

13:40–14:00 Yuichiro Toma (Nagoya University)

On the order estimation of the double L-function

The double zeta-function and double L-functions are generalizations of the classical Riemann zeta-function and L-functions to functions of two variables, respectively. Analytic properties of the double zeta-function have been studied for the last two decades. In this talk, we report a result on upper bound estimates for the double L-function attached to two primitive Dirichlet characters.

14:05–14:35 Shota Inoue (Kanagawa University)

Moments of the Riemann zeta-function twisted by arguments

We discuss moments of the Riemann zeta-function on the critical line. This topic is one of the main topics in the analytic number theory because that is related to the Lindelöf Hypothesis, especially the distribution of prime numbers. As a celebrated work, Soundararajan proved an upper bound of moments close to a conjectural bound under the Riemann Hypothesis. Later, Najnudel considered an analog of Soundararajan's work for the argument of the Riemann zeta-function. In this talk, the speaker will give a result improving Najnudel's result. This work is an analog of Harper's and also involves a slight improvement of Harper's result.

14:50–15:20 Hirotaka Kobayashi (Nagoya University)

Mean values of the Riemann zeta function on arithmetic progressions

The mean values of the Riemann zeta function have been studied for many years. The continuous mean values has been studied for a long time, and we has enriched our understanding of it. On the other hand, our knowledge on the discrete mean values are limited. Recently, Li and Radziwiłł revealed that the twisted second moments of the Riemann zeta function over arithmetic progressions shows a notable correspondence with the analogous continuous moment. In this talk, we will give the result on the second moments over arithmetic progressions without twist.

15:35–16:05 Tomokazu Onozuka (Kyushu University)

Gregory coefficients and Hurwitz–Lerch multiple zeta functions at non-positive integer points

The Hurwitz–Lerch multiple zeta functions can be continued meromorphically to the whole space. Almost all non-positive integer points lie on the set of singularities. I will talk about asymptotic behavior of the Hurwitz–Lerch multiple zeta functions at non-positive integer points and the coefficients in them. This is a joint work with Hideki Murahara.

16:20–16:50 Masatoshi Suzuki (Tokyo Institute of Technology)

On the Hilbert space defined as the completion of Weil's hermitian form

The hermitian form on the space of smooth and compactly supported functions on the real line determined from the Weil distribution is called Weil's hermitian form in this talk. Famous Weil's criterion asserts that Weil's hermitian form is positive definite if and only if the Riemann hypothesis holds. Therefore, we can obtain a Hilbert space by completing the above space with respect to Weil's hermitian form under the Riemann hypothesis. In this talk, we report that this Hilbert space is isometrically isomorphic to a reproducing kernel Hilbert space consisting of entire functions called a de Branges space. In addition, we also state an equivalence condition for the Riemann hypothesis obtained as a by-product of constructing this isomorphism.

Thursday, October 12

09:20–10:20 Gautam Chinta (City University of New York, City College)

Planes in $\mathbb{Z}^4$

We discuss the number of two-dimensional sublattices of $\mathbb{Z}^4$ of a fixed covolume and construct the associated Dirichlet series. The latter is shown to be related to Eisenstein series on both GL_4 and its metaplectic double cover. This is a joint work with Valdir Pereira Jr.

10:35–11:05 Masahiro Mine (Waseda University)

A weak form of strong universality for the Hurwitz zeta-function

Voronin's universality theorem states that non-vanishing functions can be approximated by some shifts of the Riemann zeta-function. Furthermore, the Hurwitz zeta-function has a strong universality property if the parameter is transcendental or rational in the sense that the target functions may have zeros. Note that it is an open problem whether the strong universality is valid for the Hurwitz zeta-function even if the parameter is algebraic and irrational. In this talk, I will present a weak form of strong universality for the Hurwitz zeta-function with algebraic irrational parameter. The key idea for the proof is using two random models related to the Hurwitz zeta-function, one of which is easy to deal with.

11:20–11:40 Karin Ikeda (Kyushu University)

On real zeros of the Hurwitz zeta function

In this talk, I will present a solution to the problem of real zeros of the Hurwitz zeta function, which has remained unsolved in previous studies. After reviewing the works of Spira, Nakamura, Matsusaka, and Endo-Suzuki, I discuss the remaining case, namely the zeros in the interval $(-4, 0)$. This work shows that all real zeros of the Hurwitz zeta function, like the Riemann zeta function, are simple. I also present an observation of a curious behavior of a family of polynomials used in the proof.

11:45–12:15 Masanori Katsurada (Keio University)

An application to Mellin–Barnes type integrals to some mean squares of Dirichlet–Hurwitz–Lerch L-functions

Throughout the talk, $s = \sigma + it$ is a complex variable, α and λ real parameters with $\alpha \geq 0$, χ any Dirichlet character modulo (arbitrary) $q \geq 1$, $e(s) = e^{2\pi is}$, $e_q(s) = e(s/q) = e^{2\pi is/q}$, $\varphi(q)$ Euler's totient function, and $\chi_c(l) = \chi(c+l)$ for any $c, l \in \mathbb{Z}$ a (shifted) character. The Dirichlet-Hurwitz-Lerch L-function $L_{\chi_c}(s, \alpha, \lambda)$ (attached to χ_c) is defined by

$$L_{\chi_c}(s, \alpha, \lambda) = {\sum_{l=0}^{\infty}}' \frac{\chi_c(l) e_q\{(\alpha + l)\lambda\}}{(\alpha + l)^s} \qquad (\sigma = \operatorname{Re} s > 1), \tag{1}$$

and its meromorphic continuation over the whole s-plane, where the primed summation symbol indicates omission of the impossible term $1/0^s$; this reduces if $(q, \chi) = (1, \iota)$ to the Lerch zeta-function $\psi(s, \alpha, \lambda)$, while if $(\alpha, \lambda) = (0, 0)$ to the (shifted) Dirichlet L-function $L_{\chi_c}(s)$, where ι denotes the principal character modulo 1. We shall show in the talk that complete asymptotic expansions exist for the mean square

$$\varphi(q)^{-1} \sum_{\chi (\bmod q)} \left| L_{\chi_c}(s, \alpha, \lambda) \right|^2 \tag{2}$$

in the descending order of q as $q \to +\infty$, and also for the continuous mean square

$$\int_0^1 \left| L_{\chi_c}(s, \alpha + q\xi, \lambda) \right|^2 d\xi \tag{3}$$

in the descending order of $\operatorname{Im} s = t$ as $t \to \pm\infty$. Asymptotics for some important cases on the critical lines $\sigma = 1/2$ and $\sigma = 1$ for (2) and (3) respectively, as well as at the point $s = 1$ for (2) (the principal character case $\chi = \iota$ is to be excluded in the averaging) are also treated as their limiting cases.

13:40–14:00 Kohei Takehira (Tohoku University)

On the number of points with bounded dynamical canonical height

The concept of the height function plays a fundamental role in number theory, serving as a measure of "arithmetic complexity". It is not only of technical significance but also an intriguing subject of study in its own right. For instance, Schanuel (1979) derived an asymptotic formula for the count of points in projective n-space over a number field K with bounded height, including crucial arithmetic invariants such as the class number h_K. Call-Silverman (1993) introduced the dynamical variant of height functions known as the dynamical canonical height. Hsia (1997) investigated the counting problem of points with bounded dynamical canonical height, using the dynamical height zeta function. Our research aims to provide more precise formulas for the counting problem of points with bounded dynamical canonical height in specific cases. The discussion relies on the explicit computation of the dynamical height zeta function and establishing statements in analytic number theory.

14:05–14:35 Kota Saito (University of Tsukuba)

Finiteness of solutions to linear Diophantine equations on Piatetski–Shapiro sequences

A sequence of positive integers of the form $\lfloor n^\alpha \rfloor$ for some fixed non-integral $\alpha > 1$ is called a Piatetski-Shapiro sequence. Let $\mathrm{PS}(\alpha)$ be the set of $\lfloor n^\alpha \rfloor$ for $n = 1, 2, \ldots$. In this talk, we discuss the linear equation (E) $x + y = z$ for $(x, y, z) \in \mathrm{PS}(\alpha)^3$. As a main result, we show that for almost all $\alpha > 3$, the equation (E) has at most finitely many solutions (x, y, z) which belong to $\mathrm{PS}(\alpha)^3$. If time permits, we give a result for general linear equations and the Hausdorff dimension.

14:50–15:20 Yasuhiro Ishitsuka (Kyushu University)

Exponential sums on binary quartics and its application

On 2020, Taniguchi and Thorne gave a lower bound for the number of cubic fields whose discriminants are squarefree and has at most three prime factors. A key point is to determine an exponential sum on the space of binary cubic forms.

In this talk, I introduce an analogue result on the space of binary quartic forms, and introduce its application on elliptic curves. This is based on a joint work with Takashi Taniguchi (Kobe), Frank Thorne (South Carolina) and Stanley Yao Xiao (UNBC).

15:35–16:05 Yusuke Tsuda (University of Tsukuba)

Gaps between prime numbers that satisfy the Goldbach equation

Let N be an even integer and p be a prime number less than N. If $N-p$ is prime, then we say that p satisfies the Goldbach equation for N. In this talk, we consider the gaps between prime numbers p and p' that satisfy the Goldbach equation for some N. In particular, we prove that, for any $\varepsilon > 0$, there exist such primes p and p' with

$$|p-p'| \le \left(\frac{20\sqrt{3}+15\sqrt{5}}{48\sqrt{3}} + \varepsilon \right) \mathfrak{S}(N)^{-1} (\log N)^2$$

for almost all even integers N where $\mathfrak{S}(N)$ is the singular series for the Goldbach conjecture.

16:20–16:50 Ade Irma Suriajaya (Kyushu University)

The average number of Goldbach representations and zero-free regions of the Riemann zeta-function

We prove an unconditional form of Fujii's formula for the average number of Goldbach representations and show that the error in this formula is determined by a general zero-free region of the Riemann zeta-function, and vice versa. In particular, we describe the error in the unconditional formula in terms of the remainder in the Prime Number Theorem which connects the error to zero-free regions of the Riemann zeta-function. This is joint work with Keith Billington, Maddie Cheng, and Jordan Schettler from San Jose State University.

Friday, October 13

09:30–10:30 Andrew Booker (University of Bristol)

All about murmurations

Last year, using machine learning, He, Lee, Oliver, and Pozdnyakov discovered a surprising correlation between the root numbers of elliptic curves and their a_p values for p in certain ranges, which they dubbed "murmurations of elliptic curves". Follow-up work has shown that this is a general feature present in many families of L-functions. I will briefly explain this phenomenon before presenting some joint work with Min Lee and Jonathan Bober demonstrating murmurations in an archimedean family.

10:45–11:15 Min Lee (University of Bristol)

Selberg type trace formula for automorphic forms of weight 1

The knowledge about the weight 0 automorphic forms has been accumulated greatly for the last 80 years (after Maass' paper on the Maass forms). Many mysteries still remain unsolved, but we have developed several methods to study them; one of them is the trace formula for Maass forms - Selberg trace formulas and the Bruggman-Kuznetsov trace formulas. How about the weight 1 case? Surprisingly the literature is shallow for the weight 1 automorphic forms. In this talk, we will see trace formulas for the weight 1 automorphic forms and their potential applications. This is an ongoing joint project with Andrew R. Booker.

11:30–12:00 Takafumi Miyazaki (Gunma University)

Number of solutions to a special type of Pillai's equation

A main case of the famous conjecture of S. S. Pillai states that for each positive integer c the equation $a^x - b^y = c$ has at most finitely many solutions a, b, x and y in positive integers greater than 1. Pillai's pioneer works were mainly on the case where the bases a and b are fixed and relatively prime. Under this restriction many other researchers have obtained important results, including Stroker & Tijdeman (1985), Le (1992), Scott (1993), Terai (1999), Bennett (2001, 2003), Luca (2001), Scott & Styer (2004, 2006) and Bugeaud & Luca (2006). One of the most important unsolved problems in this field is (a main case of) the conjecture put forward by Bennett in 2001 which asserts that for any fixed relatively prime positive integers a, b and c with $a > 1, b > 1$ not perfect powers there is at most one solution to the equation $a^x - b^y = c$ in positive integers x and y, except for $(a, b, c) = (2, 3, 5)$, $(2, 3, 13)$, $(2, 5, 3)$, $(3, 2, 1)$, $(13, 3, 10)$ or $(91, 2, 89)$. The main result of this talk says that for each a his conjecture holds true except for only finitely many pairs of b and c. The proof relies on the m-adic form of Baker's theory due to Bugeaud and Schmidt Subspace Theorem. This is a joint work with István Pink (University of Debrecen).

13:40–14:00 Yuta Kadono (Tohoku University)

On an integral representation of Schur type multiple polylogarithms

It is well-known that the Schur multiple zeta values (Schur MZVs), introduced by Nakasuji, Phuksuwan, and Yamasaki, satisfy the Jacobi-Trudi formula. Furthermore, Bachmann extends this result for the interpolated Schur MZVs. In this talk, we present the Jacobi-Trudi type formula for the verticallity interpolated Schur type multiple polylogarithms (Schur t-MPLs). The key aspect of the proof involves considering integrals over the square region and utilizing them to derive an integral representation for the Schur t-MPLs. Besides, we generalize these integrals to provide integral representations not only for the Schur t-MPLs but also for a certain interpolated Hurwitz type MPLs at level N and Kawashima functions.

14:05–14:35 Kyosuke Nishibiro (Tokyo Metropolitan University)

On generalization of duality formulas for the Arakawa-Kaneko type zeta functions

Kaneko and Tsumura introduced the Arakawa-Kaneko type zeta function $\eta(-k_1, \dots, -k_r; s_1, \dots, s_r)$ for non-negative integers $k_1, \dots, k_r$ and complex variables $s_1, \dots, s_r$. Recently, Yamamoto showed that, by using the multiple integral expression, $\eta(u_1, \dots, u_r; s_1, \dots, s_r)$ can be extended to an analytic function of $2r$ variables. Also, he showed that the function $\eta(u_1, \dots, u_r; s_1, \dots, s_r)$ satisfies a duality formula. In this talk, by using the non-strict multi-indexed polylogarithm, we define a kind of Arakawa-Kaneko type zeta function, and show that the function satisfies a certain duality formula.

14:50–15:20 Hideto Iwata (Nagoya University)

On some analytic properties of a function associated with the Selberg class satisfying certain special conditions

In 2001, M.Rękoś described the analytic behavior for a function connected with the Euler totient function for the upper half-plane $\mathbb{H}$. In this talk, for Im $z > 0$ we describe the analytic behavior of the generalized function $f(z, F)$, where the function F belongs to the subclass of the Selberg class which has a polynomial Euler product and satisfies some special conditions.

15:35–16:05 Shin-ichiro Seki (Aoyama Gakuin University)

A new proof of Sakugawa–Seki's and Kontsevich's functional equations via a connector

We present a new unified proof of Sakugawa–Seki's functional equation for finite multiple polylogarithms and Kontsevich's functional equation for the $1\frac{1}{2}$-logarithm using connected sums. Additionally, we plan to discuss the theory of multivariable connected sums extending this proof if time permits. This is joint work with Hanamichi Kawamura and Takumi Maesaka.