Title. Diophantine Approximation for Subvarieties and GCD problems

Abstract.

In their recent Invent. Math. paper, McKinnon and Roth introduced the approximation constant $\alpha_x(L)$ to an algebraic point x on an algebraic variety V with an ample line bundle bundle L. The invariant $\alpha_x(L)$ measures how well x can be approximated by rational points on X with respect to the height function associate to L. They showed that $\alpha_x(L)$ is closely related to the Seshadri constant $\epsilon_x(L)$ measuring the local positivity of L at x. They also showed that the invariant $\alpha_x(L)$ can be computed through another invariant $\beta_x(L)$ in the height inequality. By computing the Seshadri constant $\epsilon_x(L)$ for the case of $V = \mathbb{P}^1$, their result recovers the Roth's theorem, so the height inequality they established can be viewed as the generalization of the Roth's theorem to arbitrary projective varieties.

In my recent joint work with Min Ru, we give such results a short and simpler proof. Furthermore, we extend the results from the points of a projective variety to subschemes. This generalized result in terms of subschemes connects, as well as gives a clearer explanation, the above mentioned result of McKinnon and Roth with the recent Diophantine approximation results in term of the divisors obtained by Autissier, Corvarja, Evertse, Ferretti, Levin, Ru, Vojta, Zannier, and etc. In the first part of the talk, I will give a general survey in this direction.

We will then discuss possible application of the above result to the study of gcd problem: Let a and b be multiplicatively independent positive integers. A fundamental question is to study the non-trivial upper bound for $\gcd(a-1, b-1)$ and the asymptotic behavior of the sequence $\gcd(a^n-1, b^n-1)$. Although this approach cannot provide conclusive results for the case of number field, it does give a non-trivial estimate for the asymptotic gcd problem of entire functions since the truncated second main theorem (viewed as a higher dimensional abc theorem) is available in Nevanlinna theory. For the second part of the talk, I will discuss gcd problems for entire functions.